

解答

$$\begin{aligned}
 \boxed{1} \quad & (1) \quad x^2 + 2ax^3 + 5 - 3ax = 2ax^3 + x^2 - 3ax + 5 \\
 & (2) \quad 2x^3 + 4x^4 - a + 1 + a^2x = 4x^4 + 2x^3 + a^2x + (-a + 1) \\
 & (3) \quad x^2 + y^2 + xy + 2x + 4 = x^2 + (yx + 2x) + y^2 + 4 = x^2 + (y + 2)x + (y^2 + 4) \\
 & (4) \quad x^2 + xy + 3y + x + x^2y + 2 = (x^2 + yx^2) + (yx + x) + 3y + 2 \\
 & \quad \quad \quad = (y + 1)x^2 + (y + 1)x + (3y + 2) \\
 \boxed{2} \quad & (1) \quad 2x^3 + 2xy^2 + x^2y + 1 = 2xy^2 + x^2y + (2x^3 + 1) \\
 & (2) \quad ax^3 - x^2 + a - 1 = (x^3a + a) - x^2 - 1 = (x^3 + 1)a + (-x^2 - 1) \\
 & (3) \quad ax^3 + a^2x - 2x^2 - a^3 - 3ax^3 + 4a^3 = (4a^3 - a^3) + xa^2 + (x^3a - 3x^3a) - 2x^2 \\
 & \quad \quad \quad = 3a^3 + xa^2 - 2x^3a - 2x^2 \\
 \boxed{3} \quad & (1) \quad A + B = (5x^2 + 2x + 1) + (2x^2 + 3x + 1) = 5x^2 + 2x + 1 + 2x^2 + 3x + 1 \\
 & \quad \quad \quad = (5x^2 + 2x^2) + (2x + 3x) + (1 + 1) \\
 & \quad \quad \quad = 7x^2 + 5x + 2 \\
 & \quad \quad A - B = (5x^2 + 2x + 1) - (2x^2 + 3x + 1) = 5x^2 + 2x + 1 - 2x^2 - 3x - 1 \\
 & \quad \quad \quad = (5x^2 - 2x^2) + (2x - 3x) + (1 - 1) \\
 & \quad \quad \quad = 3x^2 - x \\
 & (2) \quad A + B = (3x + 5 - x^2) + (8x - x^2 + 5) = 3x + 5 - x^2 + 8x - x^2 + 5 \\
 & \quad \quad \quad = (-x^2 - x^2) + (3x + 8x) + (5 + 5) \\
 & \quad \quad \quad = -2x^2 + 11x + 10 \\
 & \quad \quad A - B = (3x + 5 - x^2) - (8x - x^2 + 5) = 3x + 5 - x^2 - 8x + x^2 - 5 \\
 & \quad \quad \quad = (-x^2 + x^2) + (3x - 8x) + (5 - 5) \\
 & \quad \quad \quad = -5x \\
 \boxed{4} \quad & (1) \quad a \times a^4 = a^{1+4} = a^5 \\
 & (2) \quad 2x \times 3x^2 = (2 \times 3) \times x^{1+2} = 6x^3 \\
 & (3) \quad 3y^2 \times (-5y^2) = \{3 \times (-5)\} \times y^{2+2} = -15y^4 \\
 & (10) \quad (-5ab)^2 \times a^3b = (-5)^2 \times a^2 \times b^2 \times a^3b = 25 \times a^{2+3} \times b^{2+1} = 25a^5b^3 \\
 & (11) \quad a^2b^3 \times (-3ab)^3 = a^2b^3 \times (-3)^3 \times a^3 \times b^3 = -27 \times a^{2+3} \times b^{3+3} = -27a^5b^6 \\
 & (12) \quad (-2ab^3)^3 \times (-2a^2b)^4 = (-2)^3 \times a^3 \times (b^3)^3 \times (-2)^4 \times (a^2)^4 \times b^4 \\
 & \quad \quad \quad = (-2)^{3+4} \times a^3 \times b^{3 \times 3} \times a^{2 \times 4} \times b^4 \\
 & \quad \quad \quad = (-2)^7 \times a^3 \times b^9 \times a^8 \times b^4 \\
 & \quad \quad \quad = -128 \times a^{3+8} \times b^{9+4} = -128a^{11}b^{13}
 \end{aligned}$$

$$\begin{aligned}
 \boxed{5} \quad & (1) \quad 2(3x - 5) = 2 \times 3x + 2 \times (-5) = 6x - 10 \\
 & (2) \quad (2x - 4) \times 3 = 2x \times 3 + (-4) \times 3 = 6x - 12 \\
 & (3) \quad -5(3x^2 - 2x) = -5 \times 3x^2 + (-5) \times (-2x) = -15x^2 + 10x \\
 & (4) \quad (2x^2 - 5x - 2) \times (-3) = 2x^2 \times (-3) + (-5x) \times (-3) + (-2) \times (-3) = -6x^2 + 15x + 6 \\
 \boxed{6} \quad & (1) \quad a(a^2 + a + 1) = a \cdot a^2 + a \cdot a + a \cdot 1 = a^3 + a^2 + a \\
 & (2) \quad (6 - x + 3x^2)x = (3x^2 - x + 6)x = 3x^2 \cdot x + (-x) \cdot x + 6 \cdot x = 3x^3 - x^2 + 6x \\
 & (3) \quad 2y^2(y + 1 - y^3) = 2y^2(-y^3 + y + 1) = 2y^2 \cdot (-y^3) + 2y^2 \cdot y + 2y^2 \cdot 1 = -2y^5 + 2y^3 + 2y^2 \\
 & (4) \quad (3x - x^2 + 8) \times 4x^3 = (-x^2 + 3x + 8) \times 4x^3 = (-x^2) \cdot 4x^3 + 3x \cdot 4x^3 + 8 \cdot 4x^3 \\
 & \quad \quad \quad = -4x^5 + 12x^4 + 32x^3 \\
 & (5) \quad -3b(2b^2 + 6b + 1) = -3b \cdot 2b^2 + (-3b) \cdot 6b + (-3b) \cdot 1 = -6b^3 - 18b^2 - 3b \\
 & (6) \quad (1 - x^2 - 3x) \times (-8x) = (-x^2 - 3x + 1) \times (-8x) \\
 & \quad \quad \quad = (-x^2) \cdot (-8x) + (-3x) \cdot (-8x) + 1 \cdot (-8x) \\
 & \quad \quad \quad = 8x^3 + 24x^2 - 8x \\
 \boxed{7} \quad & (1) \quad (2x + 1)(x - 3) = 2x(x - 3) + 1 \cdot (x - 3) = 2x \cdot x + 2x \cdot (-3) + x - 3 \\
 & \quad \quad \quad = 2x^2 - 6x + x - 3 \\
 & \quad \quad \quad = 2x^2 - 5x - 3 \\
 & (2) \quad (-x + 3)(4x + 1) = -x(4x + 1) + 3(4x + 1) = -x \cdot 4x + (-x) \cdot 1 + 3 \cdot 4x + 3 \cdot 1 \\
 & \quad \quad \quad = -4x^2 - x + 12x + 3 \\
 & \quad \quad \quad = -4x^2 + 11x + 3 \\
 & (3) \quad (5x - 2)(3x - 1) = 5x(3x - 1) + (-2) \cdot (3x - 1) \\
 & \quad \quad \quad = 5x \cdot 3x + 5x \cdot (-1) + (-2) \cdot 3x + (-2) \cdot (-1) \\
 & \quad \quad \quad = 15x^2 - 5x - 6x + 2 \\
 & \quad \quad \quad = 15x^2 - 11x + 2 \\
 & (4) \quad (x + y)(2x - y) = x(2x - y) + y(2x - y) = x \cdot 2x + x \cdot (-y) + y \cdot 2x + y \cdot (-y) \\
 & \quad \quad \quad = 2x^2 - xy + 2xy - y^2 \\
 & \quad \quad \quad = 2x^2 + xy - y^2 \\
 \boxed{8} \quad & (1) \quad (x - 2)(x^2 + 3x - 4) = x(x^2 + 3x - 4) - 2(x^2 + 3x - 4) = x^3 + 3x^2 - 4x - 2x^2 - 6x + 8 \\
 & \quad \quad \quad = x^3 + x^2 - 10x + 8 \\
 & (2) \quad (x + 1)(x^2 - x + 1) = x(x^2 - x + 1) + (x^2 - x + 1) = x^3 - x^2 + x + x^2 - x + 1 \\
 & \quad \quad \quad = x^3 + 1
 \end{aligned}$$

$$\begin{aligned}
(3) \quad (3x+1)(2x^2-5x+1) &= 3x(2x^2-5x+1) + (2x^2-5x+1) \\
&= 6x^3-15x^2+3x+2x^2-5x+1 \\
&= 6x^3-13x^2-2x+1
\end{aligned}$$

$$\begin{aligned}
(4) \quad (4x-3)(3x^2-x+2) &= 4x(3x^2-x+2) - 3(3x^2-x+2) \\
&= 12x^3-4x^2+8x-9x^2+3x-6 \\
&= 12x^3-13x^2+11x-6
\end{aligned}$$

$$\boxed{9} \quad (1) \quad 3A+B=3(x^2-4x+3)+(-2x^2+2x+1)=3x^2-12x+9-2x^2+2x+1 \\
= x^2-10x+10$$

$$\begin{aligned}
(2) \quad 2A-3B &= 2(x^2-4x+3)-3(-2x^2+2x+1)=2x^2-8x+6+6x^2-6x-3 \\
&= 8x^2-14x+3
\end{aligned}$$

$$\begin{aligned}
(3) \quad A+2(2A-B) &= A+4A-2B=5A-2B \\
&= 5(x^2-4x+3)-2(-2x^2+2x+1) \\
&= 5x^2-20x+15+4x^2-4x-2 \\
&= 9x^2-24x+13
\end{aligned}$$

$$\begin{aligned}
(4) \quad -5A+3B-2(-3A+4B) &= -5A+3B+6A-8B=A-5B \\
&= x^2-4x+3-5(-2x^2+2x+1) \\
&= x^2-4x+3+10x^2-10x-5 \\
&= 11x^2-14x-2
\end{aligned}$$

$$\boxed{10} \quad (1) \quad (x+1)^2=x^2+2 \cdot x \cdot 1+1^2=x^2+2x+1$$

$$(2) \quad (x-3)^2=x^2-2 \cdot x \cdot 3+3^2=x^2-6x+9$$

$$(3) \quad (2x+1)^2=(2x)^2+2 \cdot 2x \cdot 1+1^2=4x^2+4x+1$$

$$(4) \quad (3x-4)^2=(3x)^2-2 \cdot 3x \cdot 4+4^2=9x^2-24x+16$$

$$(5) \quad (3a+2b)^2=(3a)^2+2 \cdot 3a \cdot 2b+(2b)^2=9a^2+12ab+4b^2$$

$$(6) \quad (5x-2y)^2=(5x)^2-2 \cdot 5x \cdot 2y+(2y)^2=25x^2-20xy+4y^2$$

$$\boxed{11} \quad (1) \quad (x+1)(x-1)=x^2-1^2=x^2-1$$

$$(2) \quad (2x+1)(2x-1)=(2x)^2-1^2=4x^2-1$$

$$(3) \quad (1-3x)(1+3x)=1^2-(3x)^2=1-9x^2$$

$$(4) \quad (y-2x)(y+2x)=y^2-(2x)^2=y^2-4x^2$$

$$(5) \quad (a+b)(b-a)=-(a+b)(a-b)=-(a^2-b^2)=-a^2+b^2$$

$$(6) \quad (2b-3a)(3a+2b)=-(3a-2b)(3a+2b)=-\{(3a)^2-(2b)^2\}$$

$$=-(9a^2-4b^2)=-9a^2+4b^2$$

$$\boxed{12} \quad (1) \quad (x+1)(x+2)=x^2+(1+2)x+1 \cdot 2=x^2+3x+2$$

$$(2) \quad (x+3)(x+2)=x^2+(3+2)x+3 \cdot 2=x^2+5x+6$$

$$(3) \quad (x+1)(x-3)=x^2+(1-3)x+1 \cdot (-3)=x^2-2x-3$$

$$(4) \quad (x-2)(x+4)=x^2+(-2+4)x+(-2) \cdot 4=x^2+2x-8$$

$$(5) \quad (x-6)(x-5)=x^2+(-6-5)x+(-6) \cdot (-5)=x^2-11x+30$$

$$(6) \quad (x-3)(x-8)=x^2+(-3-8)x+(-3) \cdot (-8)=x^2-11x+24$$

$$\boxed{13} \quad (1) \quad (2x+1)(3x+2)=2 \cdot 3x^2+(2 \cdot 2+1 \cdot 3)x+1 \cdot 2=6x^2+7x+2$$

$$(2) \quad (4x+3)(6x+1)=4 \cdot 6x^2+(4 \cdot 1+3 \cdot 6)x+3 \cdot 1=24x^2+22x+3$$

$$(3) \quad (3x-5)(4x+1)=3 \cdot 4x^2+\{3 \cdot 1+(-5) \cdot 4\}x+(-5) \cdot 1=12x^2-17x-5$$

$$(10) \quad (5x-y)(3x+4y)=5 \cdot 3x^2+\{5 \cdot 4+(-1) \cdot 3\}xy+(-1) \cdot 4y^2=15x^2+17xy-4y^2$$

$$(11) \quad (2x+7y)(3x-2y)=2 \cdot 3x^2+\{2 \cdot (-2)+7 \cdot 3\}xy+7 \cdot (-2)y^2=6x^2+17xy-14y^2$$

$$(12) \quad (4x+3y)(2x-5y)=4 \cdot 2x^2+\{4 \cdot (-5)+3 \cdot 2\}xy+3 \cdot (-5)y^2=8x^2-14xy-15y^2$$

$$\boxed{14} \quad (1) \quad (x^2+2)^2=(x^2)^2+2 \cdot x^2 \cdot 2+2^2=x^4+4x^2+4$$

$$(2) \quad (x^2-y^2)^2=(x^2)^2-2 \cdot x^2 \cdot y^2+(y^2)^2=x^4-2x^2y^2+y^4$$

$$(3) \quad (x^3+3y^2)(x^3-3y^2)=(x^3)^2-(3y^2)^2=x^6-9y^4$$

$$(4) \quad (y^2+5)(y^2-2)=(y^2)^2+(5-2)y^2+5 \cdot (-2)=y^4+3y^2-10$$

$$\boxed{15} \quad (1) \quad a-b=A \quad \text{とおくと}$$

$$\begin{aligned}
(a-b+c)^2 &= (A+c)^2=A^2+2Ac+c^2=(a-b)^2+2(a-b)c+c^2 \\
&= (a^2-2ab+b^2)+(2ac-2bc)+c^2 \\
&= a^2+b^2+c^2-2ab-2bc+2ca
\end{aligned}$$

$$(2) \quad -a+b=A \quad \text{とおくと}$$

$$\begin{aligned}
(-a+b+c)^2 &= (A+c)^2=A^2+2Ac+c^2=(-a+b)^2+2(-a+b)c+c^2 \\
&= (a^2-2ab+b^2)+(-2ac+2bc)+c^2 \\
&= a^2+b^2+c^2-2ab+2bc-2ca
\end{aligned}$$

$$(3) \quad x+y=A \quad \text{とおくと}$$

$$\begin{aligned}
(x+y+1)^2 &= (A+1)^2=A^2+2A+1=(x+y)^2+2(x+y)+1 \\
&= (x^2+2xy+y^2)+(2x+2y)+1 \\
&= x^2+2xy+y^2+2x+2y+1
\end{aligned}$$

$$\boxed{16} \quad (1) \quad x+y=A \text{ とおくと}$$

$$\begin{aligned} (x+y+z)(x+y-z) &= (A+z)(A-z) = A^2 - z^2 \\ &= (x+y)^2 - z^2 \\ &= x^2 + 2xy + y^2 - z^2 \\ &= x^2 + y^2 - z^2 + 2xy \end{aligned}$$

$$(2) \quad x+y=A \text{ とおくと}$$

$$\begin{aligned} (x+y+1)(x+y+4) &= (A+1)(A+4) = A^2 + 5A + 4 \\ &= (x+y)^2 + 5(x+y) + 4 \\ &= (x^2 + 2xy + y^2) + (5x + 5y) + 4 \\ &= x^2 + 2xy + y^2 + 5x + 5y + 4 \end{aligned}$$

$$(3) \quad a-2b=A \text{ とおくと}$$

$$\begin{aligned} (a-2b+3c)(a-2b-3c) &= (A+3c)(A-3c) = A^2 - 9c^2 \\ &= (a-2b)^2 - 9c^2 \\ &= a^2 - 4ab + 4b^2 - 9c^2 \\ &= a^2 + 4b^2 - 9c^2 - 4ab \end{aligned}$$

$$(4) \quad 2a-b=A \text{ とおくと}$$

$$\begin{aligned} (2a-b+3)(2a-b-2) &= (A+3)(A-2) = A^2 + A - 6 \\ &= (2a-b)^2 + (2a-b) - 6 \\ &= (4a^2 - 4ab + b^2) + 2a - b - 6 \\ &= 4a^2 - 4ab + b^2 + 2a - b - 6 \end{aligned}$$

$$\boxed{17} \quad (1) \quad x^2+x=A \text{ とおくと}$$

$$\begin{aligned} (x^2+x+1)(x^2+x-1) &= (A+1)(A-1) = A^2 - 1 = (x^2+x)^2 - 1 \\ &= (x^2)^2 + 2 \cdot x^2 \cdot x + x^2 - 1 \\ &= x^4 + 2x^3 + x^2 - 1 \end{aligned}$$

$$(2) \quad x^2+2x=A \text{ とおくと}$$

$$\begin{aligned} (x^2+2x+3)(x^2+2x-1) &= (A+3)(A-1) = A^2 + 2A - 3 \\ &= (x^2+2x)^2 + 2(x^2+2x) - 3 \\ &= \{(x^2)^2 + 2 \cdot x^2 \cdot 2x + (2x)^2\} + (2x^2 + 4x) - 3 \\ &= (x^4 + 4x^3 + 4x^2) + 2x^2 + 4x - 3 \\ &= x^4 + 4x^3 + 6x^2 + 4x - 3 \end{aligned}$$

$$\boxed{18} \quad (1) \quad (x^2+9)(x+3)(x-3) = (x^2+9)\{(x+3)(x-3)\} = (x^2+9)(x^2-9) = (x^2)^2 - 9^2 = x^4 - 81$$

$$\begin{aligned} (2) \quad (4x^2+1)(2x-1)(2x+1) &= (4x^2+1)\{(2x-1)(2x+1)\} = (4x^2+1)(4x^2-1) \\ &= (4x^2)^2 - 1^2 = 16x^4 - 1 \end{aligned}$$

$$(3) \quad (x+2)^2(x-2)^2 = \{(x+2)(x-2)\}^2 = (x^2-4)^2 = (x^2)^2 - 8x^2 + 16 = x^4 - 8x^2 + 16$$

$$\begin{aligned} (4) \quad (2x+y)^2(2x-y)^2 &= \{(2x+y)(2x-y)\}^2 = \{(2x)^2 - y^2\}^2 \\ &= (4x^2 - y^2)^2 = (4x^2)^2 - 8x^2y^2 + (y^2)^2 \\ &= 16x^4 - 8x^2y^2 + y^4 \end{aligned}$$

$$\boxed{19} \quad (1) \quad mn+m=m \cdot n+m \cdot 1=m(n+1)$$

$$(2) \quad p^2q^3+pq=pq \cdot pq^2+pq \cdot 1=pq(pq^2+1)$$

$$(3) \quad ab+ac-ad=a \cdot b+a \cdot c-a \cdot d=a(b+c-d)$$

$$(4) \quad 2xy+3x^2y+6y^3=y \cdot 2x+y \cdot 3x^2+y \cdot 6y^2=y(2x+3x^2+6y^2)=y(3x^2+6y^2+2x)$$

$$\boxed{20} \quad (1) \quad x(x+1)+(x+1)=x(x+1)+1 \cdot (x+1)=(x+1)(x+1)=(x+1)^2$$

$$(2) \quad (a-1)x-(a-1)=(a-1)x-(a-1) \cdot 1=(a-1)(x-1)$$

$$(3) \quad y(x-3y)+3x(3y-x)=y(x-3y)-3x(x-3y)=(x-3y)(y-3x)$$

$$\begin{aligned} (4) \quad 5x^2(2x-1)-2x(1-2x) &= 5x^2(2x-1)+2x(2x-1)=5x \cdot x(2x-1)+2 \cdot x(2x-1) \\ &= x(2x-1)(5x+2) \end{aligned}$$

$$\boxed{21} \quad (1) \quad x^2+2x+1=x^2+2 \cdot x \cdot 1+1^2=(x+1)^2$$

$$(2) \quad x^2-4x+4=x^2-2 \cdot x \cdot 2+2^2=(x-2)^2$$

$$(7) \quad 4a^2+4ab+b^2=(2a)^2+2 \cdot 2a \cdot b+b^2=(2a+b)^2$$

$$(8) \quad 9a^2-12ab+4b^2=(3a)^2-2 \cdot 3a \cdot 2b+(2b)^2=(3a-2b)^2$$

$$\boxed{22} \quad (1) \quad a^2-1=a^2-1^2=(a+1)(a-1)$$

$$(2) \quad 4a^2-9=(2a)^2-3^2=(2a+3)(2a-3)$$

$$(3) \quad a^2-16b^2=a^2-(4b)^2=(a+4b)(a-4b)$$

$$(4) \quad b^2-36a^2=b^2-(6a)^2=(b+6a)(b-6a)$$

$$\boxed{23} \quad (1) \quad x^2+4x+3=x^2+(1+3)x+1 \cdot 3=(x+1)(x+3)$$

$$(2) \quad x^2+6x+5=x^2+(1+5)x+1 \cdot 5=(x+1)(x+5)$$

$$(3) \quad x^2+5x+6=x^2+(2+3)x+2 \cdot 3=(x+2)(x+3)$$

$$(4) \quad x^2+7x+6=x^2+(1+6)x+1 \cdot 6=(x+1)(x+6)$$

$$(5) \quad x^2+7x+12=x^2+(3+4)x+3 \cdot 4=(x+3)(x+4)$$

$$(6) \quad x^2+9x+14=x^2+(2+7)x+2 \cdot 7=(x+2)(x+7)$$

[24] (1) $x^2 + 6xy + 8y^2 = x^2 + (2y + 4y)x + 2y \cdot 4y = (x + 2y)(x + 4y)$
 (2) $x^2 + 9xy + 8y^2 = x^2 + (y + 8y)x + y \cdot 8y = (x + y)(x + 8y)$
 (3) $x^2 + 5xy - 6y^2 = x^2 + (-y + 6y)x + (-y) \cdot 6y = (x - y)(x + 6y)$
 (4) $x^2 - 2xy - 15y^2 = x^2 + (3y - 5y)x + 3y \cdot (-5y) = (x + 3y)(x - 5y)$

[25] (1) $6x^2 + 5x + 1 = (2x + 1)(3x + 1)$
 (2) $6x^2 + 7x + 1 = (x + 1)(6x + 1)$

(1)

$$\begin{array}{r} 2 \quad 1 \longrightarrow 3 \\ 3 \quad 1 \longrightarrow 2 \\ \hline 6 \quad 1 \quad 5 \end{array}$$

(2)

$$\begin{array}{r} 1 \quad 1 \longrightarrow 6 \\ 6 \quad 1 \longrightarrow 1 \\ \hline 6 \quad 1 \quad 7 \end{array}$$

(3) $12x^2 + 7x + 1 = (3x + 1)(4x + 1)$
 (4) $12x^2 + 8x + 1 = (2x + 1)(6x + 1)$

(3)

$$\begin{array}{r} 3 \quad 1 \longrightarrow 4 \\ 4 \quad 1 \longrightarrow 3 \\ \hline 12 \quad 1 \quad 7 \end{array}$$

(4)

$$\begin{array}{r} 2 \quad 1 \longrightarrow 6 \\ 6 \quad 1 \longrightarrow 2 \\ \hline 12 \quad 1 \quad 8 \end{array}$$

[26] (1) $3x^2 + 4xy + y^2 = (x + y)(3x + y)$
 (2) $6x^2 + 7xy + 2y^2 = (2x + y)(3x + 2y)$

(1)

$$\begin{array}{r} 1 \quad y \longrightarrow 3y \\ 3 \quad y \longrightarrow y \\ \hline 3 \quad y^2 \quad 4y \end{array}$$

(2)

$$\begin{array}{r} 2 \quad y \longrightarrow 3y \\ 3 \quad 2y \longrightarrow 4y \\ \hline 6 \quad 2y^2 \quad 7y \end{array}$$

(3) $2x^2 + xy - 6y^2 = (x + 2y)(2x - 3y)$
 (4) $3x^2 - 5xy + 2y^2 = (x - y)(3x - 2y)$

(3)

$$\begin{array}{r} 1 \quad 2y \longrightarrow 4y \\ 2 \quad -3y \longrightarrow -3y \\ \hline 2 \quad -6y^2 \quad y \end{array}$$

(4)

$$\begin{array}{r} 1 \quad -y \longrightarrow -3y \\ 3 \quad -2y \longrightarrow -2y \\ \hline 3 \quad 2y^2 \quad -5y \end{array}$$

[27] (1) $2x^2 - 2y^2 = 2(x^2 - y^2) = 2(x + y)(x - y)$
 (2) $3x^2 - 9x + 6 = 3(x^2 - 3x + 2) = 3\{x^2 + (-1 - 2)x + (-1) \cdot (-2)\} = 3(x - 1)(x - 2)$
 (3) $x^2y - 10xy + 25y = y(x^2 - 10x + 25) = y(x^2 - 2 \cdot x \cdot 5 + 5^2) = y(x - 5)^2$
 (4) $x^3y - 3x^2y + 2xy = xy(x^2 - 3x + 2) = xy\{x^2 + (-1 - 2)x + (-1) \cdot (-2)\}$

$$= xy(x - 1)(x - 2)$$

[28] (1) $x - y = A$ とおくと

$$(x - y)^2 + 3(x - y) = A^2 + 3A = A(A + 3) = (x - y)(x - y + 3)$$

 (2) $(5x + 3)^2 - 5x - 3 = (5x + 3)^2 - (5x + 3)$
 よって, $5x + 3 = A$ とおくと

$$(5x + 3)^2 - 5x - 3 = A^2 - A = A(A - 1) = (5x + 3)\{(5x + 3) - 1\} = (5x + 2)(5x + 3)$$

 (5) $xy = A$ とおくと

$$(xy)^2 + 3xy + 2 = A^2 + 3A + 2 = (A + 1)(A + 2) = (xy + 1)(xy + 2)$$

 (6) $xy = A$ とおくと

$$x^2y^2 - 4xy + 3 = (xy)^2 - 4xy + 3 = A^2 - 4A + 3 = (A - 1)(A - 3) = (xy - 1)(xy - 3)$$

[29] (1) $(x - 1)^2 - y^2 = \{(x - 1) + y\}\{(x - 1) - y\} = (x + y - 1)(x - y - 1)$
 (2) $(x + y)^2 - 9 = \{(x + y) + 3\}\{(x + y) - 3\} = (x + y + 3)(x + y - 3)$
 (5) $x^2 + 2x + 1 - 4y^2 = (x^2 + 2x + 1) - 4y^2 = (x + 1)^2 - 4y^2$

$$= \{(x + 1) + 2y\}\{(x + 1) - 2y\} = (x + 2y + 1)(x - 2y + 1)$$

 (6) $x^2 - 6xy + 9y^2 - 1 = (x^2 - 6xy + 9y^2) - 1 = (x - 3y)^2 - 1$

$$= \{(x - 3y) + 1\}\{(x - 3y) - 1\}$$

$$= (x - 3y + 1)(x - 3y - 1)$$

[30] (1) $a^2 + ab - 2a + b - 3 = (a + 1)b + (a^2 - 2a - 3) = (a + 1)b + (a + 1)(a - 3)$

$$= (a + 1)\{b + (a - 3)\} = (a + 1)(a + b - 3)$$

 (2) $a^2 + 2ab + 4b + 3a + 2 = (2a + 4)b + (a^2 + 3a + 2) = 2(a + 2)b + (a + 1)(a + 2)$

$$= (a + 2)\{2b + (a + 1)\} = (a + 2)(a + 2b + 1)$$

[31] (1) $x^2 + (2y + 3)x + (y + 1)(y + 2) = \{x + (y + 1)\}\{x + (y + 2)\} = (x + y + 1)(x + y + 2)$

$$\begin{array}{r} 1 \quad y+1 \longrightarrow y+1 \\ 1 \quad y+2 \longrightarrow y+2 \\ \hline 1 \quad (y+1)(y+2) \quad 2y+3 \end{array}$$

$$(2) \quad x^2 - x - (y-2)(y-3) = \{x - (y-2)\}\{x + (y-3)\} = (x - y + 2)(x + y - 3)$$

$$\begin{array}{rcl} 1 & \times & -(y-2) & \longrightarrow & -y+2 \\ 1 & & y-3 & \longrightarrow & y-3 \\ \hline 1 & & -(y-2)(y-3) & & -1 \end{array}$$

$$\begin{aligned} (3) \quad x^2 + (-y-1)x - (2y^2 - 5y + 2) &= x^2 + (-y-1)x - (y-2)(2y-1) \\ &= \{x + (y-2)\}\{x - (2y-1)\} \\ &= (x + y - 2)(x - 2y + 1) \end{aligned}$$

$$\begin{array}{rcl} 1 & \times & y-2 & \longrightarrow & y-2 \\ 1 & & -(2y-1) & \longrightarrow & -2y+1 \\ \hline 1 & & -(y-2)(2y-1) & & -y-1 \end{array}$$

$$\begin{aligned} (4) \quad x^2 - (5y-2)x + 6y^2 - 5y + 1 &= x^2 - (5y-2)x + (6y^2 - 5y + 1) \\ &= x^2 + (-5y+2)x + (2y-1)(3y-1) \\ &= \{x - (2y-1)\}\{x - (3y-1)\} \\ &= (x - 2y + 1)(x - 3y + 1) \end{aligned}$$

$$\begin{array}{rcl} 1 & \times & -(2y-1) & \longrightarrow & -2y+1 \\ 1 & & -(3y-1) & \longrightarrow & -3y+1 \\ \hline 1 & & (2y-1)(3y-1) & & -5y+2 \end{array}$$

$$\boxed{32} \quad (1) \quad |6| = 6$$

$$(2) \quad |-1| = 1$$

$$(3) \quad \left| -\frac{1}{8} \right| = \frac{1}{8}$$

$$(4) \quad |-4.5| = 4.5$$

$$(5) \quad |7-5| = |2| = 2$$

$$(6) \quad |-5+2| = |-3| = 3$$

$$(7) \quad |-5| + |4| = 5 + 4 = 9$$

$$(8) \quad |-2| - |-7| = 2 - 7 = -5$$

$$\boxed{33} \quad (1) \quad x=4 \text{ のとき}$$

$$|x-3| + |2x+1| = |4-3| + |2 \cdot 4 + 1| = |1| + |9| = 1 + 9 = 10$$

$$(2) \quad x=1 \text{ のとき}$$

$$|x-3| + |2x+1| = |1-3| + |2 \cdot 1 + 1| = |-2| + |3| = 2 + 3 = 5$$

$$(3) \quad x=-2 \text{ のとき}$$

$$|x-3| + |2x+1| = |-2-3| + |2 \cdot (-2) + 1| = |-5| + |-3| = 5 + 3 = 8$$

$$\boxed{34} \quad (1) \quad -\sqrt{25} = -\sqrt{5^2} = -5$$

$$(2) \quad (\sqrt{7})^2 = 7$$

$$(3) \quad (-\sqrt{6})^2 = 6$$

$$(4) \quad \sqrt{(-2)^2} = \sqrt{4} = \sqrt{2^2} = 2$$

$$\boxed{35} \quad (1) \quad \sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$$

$$(2) \quad -\sqrt{5} \times \sqrt{3} = -(\sqrt{5} \times \sqrt{3}) = -\sqrt{5 \times 3} = -\sqrt{15}$$

$$(3) \quad \sqrt{2} \times \sqrt{5} \times \sqrt{7} = \sqrt{2 \times 5 \times 7} = \sqrt{70}$$

$$(4) \quad \frac{\sqrt{35}}{\sqrt{5}} = \sqrt{\frac{35}{5}} = \sqrt{7}$$

$$(5) \quad \frac{\sqrt{39}}{\sqrt{3}} = \sqrt{\frac{39}{3}} = \sqrt{13}$$

$$(6) \quad \frac{\sqrt{14} \times \sqrt{3}}{\sqrt{6}} = \frac{\sqrt{14 \times 3}}{\sqrt{6}} = \sqrt{\frac{14 \times 3}{6}} = \sqrt{7}$$

$$\boxed{36} \quad (1) \quad \sqrt{8} = \sqrt{2^2 \times 2} = \sqrt{2^2} \times \sqrt{2} = 2\sqrt{2}$$

$$(2) \quad \sqrt{24} = \sqrt{2^2 \times 6} = \sqrt{2^2} \times \sqrt{6} = 2\sqrt{6}$$

$$(3) \quad \sqrt{32} = \sqrt{4^2 \times 2} = \sqrt{4^2} \times \sqrt{2} = 4\sqrt{2}$$

$$(4) \quad \sqrt{45} = \sqrt{3^2 \times 5} = \sqrt{3^2} \times \sqrt{5} = 3\sqrt{5}$$

$$(5) \quad \sqrt{48} = \sqrt{4^2 \times 3} = \sqrt{4^2} \times \sqrt{3} = 4\sqrt{3}$$

$$(6) \quad \sqrt{50} = \sqrt{5^2 \times 2} = \sqrt{5^2} \times \sqrt{2} = 5\sqrt{2}$$

$$\boxed{37} \quad (1) \quad \sqrt{3} \times \sqrt{15} = \sqrt{3 \times 15} = \sqrt{45} = \sqrt{3^2 \times 5} = \sqrt{3^2} \times \sqrt{5} = 3\sqrt{5}$$

$$(2) \quad \sqrt{6} \times \sqrt{14} = \sqrt{6 \times 14} = \sqrt{84} = \sqrt{2^2 \times 21} = \sqrt{2^2} \times \sqrt{21} = 2\sqrt{21}$$

$$(3) \quad \frac{\sqrt{24}}{\sqrt{3}} = \sqrt{\frac{24}{3}} = \sqrt{8} = \sqrt{2^2 \times 2} = \sqrt{2^2} \times \sqrt{2} = 2\sqrt{2}$$

$$(4) \quad \frac{\sqrt{90}}{\sqrt{5}} = \sqrt{\frac{90}{5}} = \sqrt{18} = \sqrt{3^2 \times 2} = \sqrt{3^2} \times \sqrt{2} = 3\sqrt{2}$$

$$\boxed{38} \quad (1) \quad 2\sqrt{5} + 6\sqrt{5} - 4\sqrt{5} = (2+6-4)\sqrt{5} = 4\sqrt{5}$$

$$(2) \quad 2\sqrt{5} - 4\sqrt{2} - 3\sqrt{2} + \sqrt{5} = (2+1)\sqrt{5} + (-4-3)\sqrt{2} = 3\sqrt{5} - 7\sqrt{2}$$

$$(3) \quad \sqrt{18} = \sqrt{3^2 \times 2} = 3\sqrt{2}, \quad \sqrt{50} = \sqrt{5^2 \times 2} = 5\sqrt{2} \text{ であるから}$$

$$\sqrt{18} + \sqrt{50} = 3\sqrt{2} + 5\sqrt{2} = (3+5)\sqrt{2} = 8\sqrt{2}$$

$$(4) \quad \sqrt{45} = \sqrt{3^2 \times 5} = 3\sqrt{5}, \quad \sqrt{20} = \sqrt{2^2 \times 5} = 2\sqrt{5} \quad \text{であるから}$$

$$\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = (3-2)\sqrt{5} = \sqrt{5}$$

$$\begin{aligned} [39] (1) \quad & (\sqrt{7} + \sqrt{2})(2\sqrt{7} + 3\sqrt{2}) = \sqrt{7} \times 2\sqrt{7} + \sqrt{7} \times 3\sqrt{2} + \sqrt{2} \times 2\sqrt{7} + \sqrt{2} \times 3\sqrt{2} \\ & = 2 \times (\sqrt{7})^2 + 3\sqrt{14} + 2\sqrt{14} + 3 \times (\sqrt{2})^2 \\ & = 20 + 5\sqrt{14} \end{aligned}$$

$$\begin{aligned} (2) \quad & (2\sqrt{3} + \sqrt{2})(3\sqrt{3} + 2\sqrt{2}) = 2\sqrt{3} \times 3\sqrt{3} + 2\sqrt{3} \times 2\sqrt{2} + \sqrt{2} \times 3\sqrt{3} + \sqrt{2} \times 2\sqrt{2} \\ & = 6 \times (\sqrt{3})^2 + 4\sqrt{6} + 3\sqrt{6} + 2 \times (\sqrt{2})^2 \\ & = 22 + 7\sqrt{6} \end{aligned}$$

$$\begin{aligned} (3) \quad & (3\sqrt{2} + \sqrt{6})^2 = (3\sqrt{2})^2 + 2 \times 3\sqrt{2} \times \sqrt{6} + (\sqrt{6})^2 \\ & = 9 \times (\sqrt{2})^2 + 6\sqrt{12} + 6 \\ & = 24 + 12\sqrt{3} \end{aligned}$$

$$\begin{aligned} (4) \quad & (2\sqrt{6} - \sqrt{3})^2 = (2\sqrt{6})^2 - 2 \times 2\sqrt{6} \times \sqrt{3} + (\sqrt{3})^2 \\ & = 4 \times (\sqrt{6})^2 - 4\sqrt{18} + (\sqrt{3})^2 \\ & = 27 - 12\sqrt{2} \end{aligned}$$

$$[40] (1) \quad \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{(\sqrt{5})^2} = \frac{\sqrt{5}}{5}$$

$$(2) \quad \frac{3}{\sqrt{2}} = \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{(\sqrt{2})^2} = \frac{3\sqrt{2}}{2}$$

$$(3) \quad \frac{5\sqrt{2}}{\sqrt{7}} = \frac{5\sqrt{2} \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{5\sqrt{2} \times \sqrt{7}}{(\sqrt{7})^2} = \frac{5\sqrt{14}}{7}$$

$$(4) \quad \frac{\sqrt{5}}{3\sqrt{2}} = \frac{\sqrt{5} \times \sqrt{2}}{3\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{5} \times \sqrt{2}}{3 \times (\sqrt{2})^2} = \frac{\sqrt{10}}{6}$$

$$(5) \quad \frac{1}{\sqrt{3} + \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} = \sqrt{3} - \sqrt{2}$$

$$(6) \quad \frac{\sqrt{3}}{2 - \sqrt{3}} = \frac{\sqrt{3}(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} = \frac{2\sqrt{3} + (\sqrt{3})^2}{2^2 - (\sqrt{3})^2} = 2\sqrt{3} + 3$$

$$\begin{aligned} [41] (1) \quad & (1 - \sqrt{2} + \sqrt{3})^2 = \{(1 - \sqrt{2}) + \sqrt{3}\}^2 = (1 - \sqrt{2})^2 + 2(1 - \sqrt{2})\sqrt{3} + (\sqrt{3})^2 \\ & = 1^2 - 2\sqrt{2} + (\sqrt{2})^2 + 2\sqrt{3} - 2\sqrt{6} + 3 \\ & = 6 - 2\sqrt{2} + 2\sqrt{3} - 2\sqrt{6} \end{aligned}$$

$$\text{別解} \quad (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \quad \text{を利用すると}$$

$$\begin{aligned} & (1 - \sqrt{2} + \sqrt{3})^2 \\ & = 1^2 + (-\sqrt{2})^2 + (\sqrt{3})^2 + 2 \times 1 \times (-\sqrt{2}) + 2 \times (-\sqrt{2}) \times \sqrt{3} + 2 \times \sqrt{3} \times 1 \\ & = 6 - 2\sqrt{2} + 2\sqrt{3} - 2\sqrt{6} \end{aligned}$$

$$\begin{aligned} (2) \quad & (1 + \sqrt{3} + \sqrt{5})(1 + \sqrt{3} - \sqrt{5}) = (1 + \sqrt{3})^2 - (\sqrt{5})^2 = 1^2 + 2\sqrt{3} + (\sqrt{3})^2 - 5 \\ & = 2\sqrt{3} - 1 \end{aligned}$$

$$\begin{aligned} (3) \quad & \frac{\sqrt{48}}{\sqrt{6}} + \sqrt{18} = \sqrt{\frac{48}{6}} + \sqrt{18} = \sqrt{8} + \sqrt{18} \\ & = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} \end{aligned}$$

$$(4) \quad \frac{14}{\sqrt{28}} - \sqrt{28} = \frac{14}{2\sqrt{7}} - 2\sqrt{7} = \frac{7}{\sqrt{7}} - 2\sqrt{7} = \frac{(\sqrt{7})^2}{\sqrt{7}} - 2\sqrt{7} = \sqrt{7} - 2\sqrt{7} = -\sqrt{7}$$

$$[42] (1) \quad \sqrt{3} = 1.732\cdots \quad \text{であるから} \quad 5 + \sqrt{3} = 6.732\cdots$$

$$\text{よって} \quad a = 6$$

$$\text{別解} \quad 1 < \sqrt{3} < 2 \quad \text{であるから} \quad 6 < 5 + \sqrt{3} < 7$$

$$\text{よって} \quad a = 6$$

$$(2) \quad a + b = 5 + \sqrt{3} \quad \text{であるから} \quad b = (5 + \sqrt{3}) - a = (5 + \sqrt{3}) - 6 = \sqrt{3} - 1$$

$$(3) \quad a^2 + b^2 = 6^2 + (\sqrt{3} - 1)^2 = 36 + \{(\sqrt{3})^2 - 2 \cdot \sqrt{3} \cdot 1 + 1^2\} = 36 + (4 - 2\sqrt{3}) = 40 - 2\sqrt{3}$$

$$[43] (1) \quad x + y = (3 - 2\sqrt{2}) + (3 + 2\sqrt{2}) = 6$$

$$(2) \quad xy = (3 - 2\sqrt{2})(3 + 2\sqrt{2}) = 3^2 - (2\sqrt{2})^2 = 9 - 8 = 1$$

$$(3) \quad x^2 + y^2 = (x + y)^2 - 2xy = 6^2 - 2 \cdot 1 = 36 - 2 = 34$$

$$(4) \quad x^2y + xy^2 = xy(x + y) = 1 \cdot 6 = 6$$